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Flat broadband chaos generation in a discrete-mode laser subject to optical feedback

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Abstract: Chaos generation in a discrete-mode (DM) laser subject to optical feedback is experimentally explored. The results show that a DM laser with only optical feedback can produce flat broadband chaos under an optimized feedback ratio. The effect of the laser bias current on the bandwidth and flatness of chaos is also investigated. It shows that the higher bias current, the better the flatness that can be obtained at the optimal feedback ratio.

1. Introduction

Optical chaos has many potential applications, such as secure optical communications [1-6], time domain reflectometry [7-8], physical random number generation [9-13] and lidar [14], therefore, the generation of optical chaos has attracted significant research interests [15-27]. Optical chaos generated in semiconductor lasers can be divided into two types [20]. The first type of chaos is obtained without external perturbations, for example, chaos in a free-running quantum dot vertical cavity surface emitting laser (VCSEL) due to the nonlinear coupling between two elliptical polarization modes [19]. The second type of optical chaos in semiconductor lasers is generated by adding perturbations, such as direct modulation, optical injection, or optical feedback. The latter approach is commonly used to generate chaos in semiconductor lasers by external cavity optical feedback due to its simplicity [1-4], [7-9]. However, the power spectrum of chaos generated by optical feedback is mostly not flat due to the relaxation oscillation frequency, which will affect its applications. The unflatten power spectrum of chaos reduce the randomness of the chaotic optical signal, so the distribution of random bits generated using such chaos is not symmetrical. Many techniques have been proposed and demonstrated to achieve flat chaos. Flat chaos has been achieved using electro-optic or optoelectronic oscillators [15-17]. However, these techniques are expensive because they require wideband electronic amplifiers and high-speed modulators. A flat spectrum with a large bandwidth has also been demonstrated by heterodyning two chaos [23]. All optical approach to achieve flat chaos is being sought by many groups. A flat chaos has been demonstrated in an optical feedback distributed feedback (DFB) laser using a fiber ring resonator which contains an erbium-doped fiber amplifier (EDFA) and a fiber grating [18], and this method has been extended to a VCSEL with a fiber ring resonator, which includes a semiconductor optical amplifier (SOA) and a fiber grating [21]. A very broadband flat chaos was achieved recently using a very strong feedback and long feedback delay time and the use of a highly nonlinear fiber in the feedback loop [27]. The phenomenon of a flat wideband chaos has also been observed in mutually coupled VCSELs by combining two non-uniform chaotic signals [22]. Last year, Taiyuan's group simply the experimental setups to achieve flat chaos using a band-pass filter to select a single longitudinal mode from a multi-mode laser with optical feedback [24] or mutual injection of semiconductor lasers [25].

A discrete mode (DM) laser is a special type of Fabry–Pérot (FP) lasers. Single mode operation in DM lasers is achieved by etching a small number of refractive index perturbations along the FP cavity [28]. DM lasers have many advantages, such as high bandwidth, less sensitive to optical feedback, stable single mode emission, ultra-wide temperature range of operation, low cost and ease of integration, which can be used in optical coherent communication systems, optical synthesis of terahertz wave frequencies, sensors and clocks [29-30]. However, reports on the dynamics of DM lasers are scarce [31-33]. Because the design of DM lasers has achieved enhancing one mode and suppressed the other modes, in this paper, we investigate whether flat chaos can be generated in a DM laser with filter free optical feedback.

In this study, the chaos bandwidth and its flatness are used to quantify the characteristic of chaos. Two definitions of chaos bandwidth are adopted. One is the traditional chaos bandwidth, which is the frequency between DC and the frequency which contains 80% of the power [21]. Another bandwidth is introduced as the effective bandwidth, which adds those frequency spectrum segments containing 80% of the total power in the chaos power spectrum, and such bandwidth can effectively distinguish the broadband chaotic states from the narrowband periodic states [34]. Comparing the chaotic bandwidth difference between the traditional chaos bandwidth and effective chaos bandwidth can also provide quantitative information about the flatness of chaos. The flatness of chaos in this paper is defined as the power ratio of the square root of the maximum power to the minimum power within the traditional chaos bandwidth.

2. Experimental setup

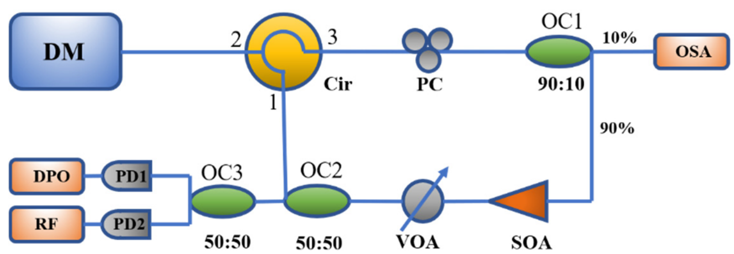

Fig. 1 Experimental setup. DM – discrete-mode laser; Cir – optical circulator; OC – fiber coupler; PC – polarization controller; SOA – semiconductor optical amplifier; VOA – variable optical attenuator; PD – photodetector; DPO – oscilloscope; RF – radio frequency spectrum analyzer; OSA – optical spectrum analyzer.

The experimental setup for the investigation of the dynamics of the DM laser with optical feedback is shown in Fig. 1. A commercial DM laser with an operating wavelength around 1550nm was used in the experiment. An extra low noise current source was used to drive the DM laser and the laser's temperature was stabilized within 0.01 K using a temperature controller. The laser's threshold current (I_{th}) at the experiment is 12.0 mA. The output of the DM laser passed through an optical circulator (Cir), a polarization controller (PC) and an optical coupler (OC1). The PC was used to control the polarization of the feedback beam to ensure that the maximum effect of the optical feedback on the dynamics of the DM laser. The OC1 split the beam into two paths. One path was connected to an optical spectrum analyzer (OSA: Agilent 86141B with a resolution of 0.06 nm) for measuring the optical spectra. Another path was linked to a semiconductor optical amplifier (SOA), a variable optical attenuator (VOA) and the second optical coupler (OC2) to form the optical feedback loop. The SOA was used to amplify the feedback power and the VOA was used to adjust the feedback power. The laser output used for detection was separated from the feedback loop using the OC2. In the detection part, the laser output was sent to a 12 GHz photodetector (PD1: New Focus 1544-B) and a 25 GHz photodetector (PD2: New Focus 1414). The output from the 25 GHz photodetector was amplified by a 20 GHz electrical amplifier and then measured by a 30 GHz bandwidth RF spectrum analyzer (RF: Anritsu MS2667C). The output of 12 GHz photodetector was sent to a 12.5 GHz bandwidth oscilloscope (DPO: Tektronix 71254C) with a sampling speed of 50 GS/s. In the experiment, the laser's bias current was fixed at 70 mA, unless stated otherwise. The feedback round trip time was about 109.06 ns.

3. Experimental results and discussion

The time trace and the power spectra of the DM laser output subject to optical feedback are illustrated in Fig. 2. The feedback ratio in Fig. 2(a) and (b) is 0.51. The ratio of the feedback power to the free running laser output power is named as the optical feedback ratio, and the feedback power is measured just before the feedback beam is fed back into the DM laser. The black and grey lines are the output of the DM laser and noise floor, respectively. A sharp increase at 8 GHz for the noise floor of the power spectrum in Fig. 2 is because the different frequency ranges in the RF spectrum analyzer use the different harmonic orders of the mixer [35]. The results in Fig. 2(a) and (b) clearly indicate that the DM is undergoing chaos dynamic with the feedback ratio of 0.51. The power spectrum in Fig. 2(b) is a common power spectrum of chaos achieved by optical feedback, where there is a peak around the laser relaxation oscillation frequency of about 9.5 GHz and the powers at the lower frequencies are low. When we increase the feedback power, the powers at lower frequencies increase. Fig. 2(c) shows the power spectrum of the DM laser output when the feedback ratio is 0.68. We can see that compared with Fig. 2(b), the powers at lower frequencies are enhanced by about 3 dB, and the power spectrum becomes flat. A further increase in the feedback ratio to 0.87, sees the power in the lower frequencies increase further by about 1 dB, as shown in Fig. 2(d), the power spectrum is no longer flat. The results in Fig. 2 clearly demonstrate that a flat broadband chaos can be easily achieved in a DM laser subject to optical feedback by properly selecting the feedback ratio.

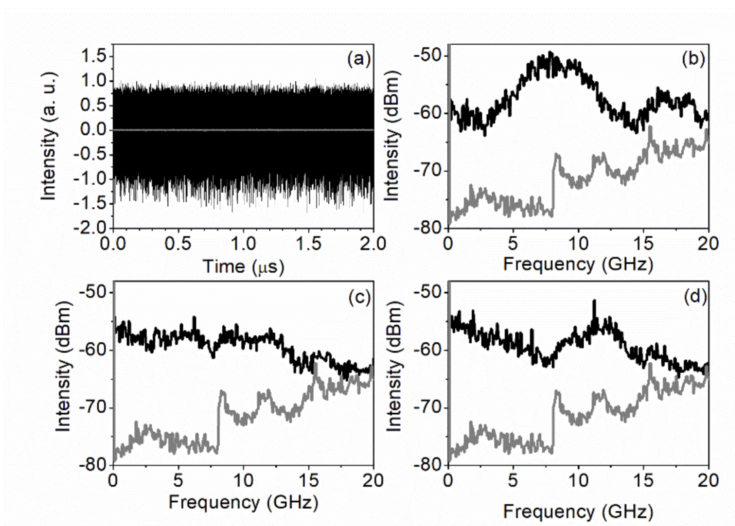

Fig.2 Time trace and power spectra of the DM laser output at the bias current of 70 mA. The feedback ratio is (a), (b) 0.51, (c) 0.68, (d) 0.87. The black and grey lines are for the laser output and noise floor, respectively.

The optical spectrum of the DM laser is also examined. The black and red curves in Fig. 3 are the optical spectra of the DM laser without and with optical feedback at the bias current of 70 mA. The feedback ratio for the red curve is 0.51. The inset of Fig. 3 is the optical spectrum of the DM laser without optical feedback in a wider wavelength range. We can see that the laser is a single longitudinal mode laser with more than 40 dB side-mode suppression and the mode spacing is about 1.25nm. The optical spectra of the DM laser under other bias currents show similar characteristics. From the results in Fig. 2, we know that the DM laser with the optical feedback in Fig. 3 undergoes chaotic dynamics and the optical spectrum clearly shows that the linewidth is broaden compared to that of the laser without optical feedback, but the DM laser still works around a single longitudinal mode, no side mode is excited. The optical spectra of the DM laser with the feedback ratio of 0.68 and 0.87 are similar to that of the DM laser with the feedback ratio of 0.51, no side mode is excited. Therefore, the etched features along the FP cavity in the DM laser act like a band-pass filter to filter out one FP mode [28], and the DM laser works like a multi-mode laser with a band-pass filter. The powers at low frequencies of chaos are enhanced by selecting only one mode and the chaos power spectrum becomes flat [24].

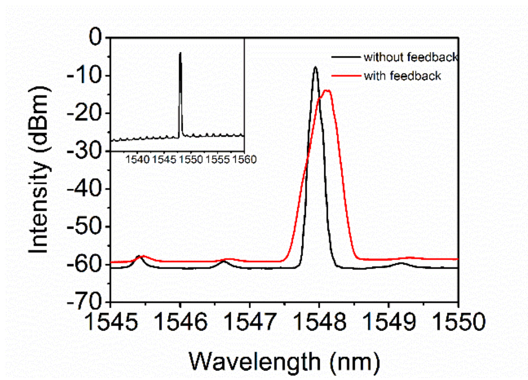

Fig. 3 Optical spectra of the DM laser output. The black curve is for the DM laser without optical feedback. The red curve is for the DM laser with optical feedback and the optical feedback ratio is 0.51. The inset is the optical spectrum of the laser without optical feedback in a wider wavelength range.

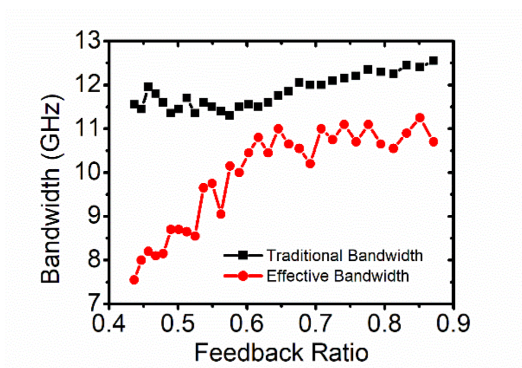

Fig. 4 The chaos bandwidth as a function of the feedback ratio. The curves with squares and circles are for the traditional bandwidth and the effective bandwidth, respectively.

Figure 4 represents the chaos bandwidth as a function of the feedback ratio. The DM laser is less sensitive to optical feedback, it needs a relatively high feedback ratio for the laser to enter a chaotic dynamic. For the laser used in this experiment, the minimum feedback ratio required to enter chaos for the laser with a bias current of 70 mA is 0.44. We also test three other DM lasers, all required a comparable feedback ratio for the lasers to exhibit chaotic dynamics. The curve with squares in Fig. 4 is based on the traditional chaos bandwidth definition. The result shows that the chaos bandwidths fluctuation around 11.5 GHz for the feedback ratio between 0.44 and 0.62. When the feedback ratio is greater than 0.62, the traditional chaos bandwidths start to increase slowly as the feedback ratio increases. The curve with circles is based on the effective bandwidth definition. The result shows that the effective bandwidth is about 4 GHz lower than the traditional bandwidth for the feedback ratio of 0.44.

For feedback ratios between 0.44 and 0.64, the improvement in the effective bandwidth with the increasing feedback ratio is much faster than that of the traditional bandwidth. This is because the power spectra become flattener with the increase of the feedback ratio (shown in Fig. 5). Further increasing the feedback ratio, the effective bandwidth shows a plateau. This is due to the increase in traditional chaos bandwidth and uneven power spectra.

Figure 5 illustrates chaos flatness as a function of the feedback ratio. The result shows that the chaos spectrum first becomes flatter as the feedback ratio increases, and the flatness drops from about 8.5 dB to 2.8 dB. When the feedback ratios are between 0.64 and 0.68, the flatness remains at around 3 dB. Further increases the feedback ratio see the flatness starts to increase again. This is because the power in the low-frequency components is further enhanced. Figs. 4 and 5 demonstrate that a DM laser with only optical feedback can generate a flat broadband chaos with the traditional bandwidth of 12.1 GHz and effective bandwidth of 10.6 GHz and flatness of 2.8 dB. This bandwidth is wider than that of the optical feedback multi-mode laser with a band-pass filter [24].

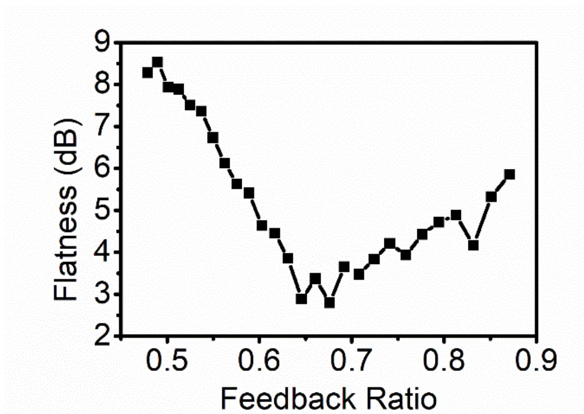

Fig.5 The flatness of the chaos as a function of the feedback ratio.

In the following, we study the effect of the bias current on the chaos bandwidth and flatness. Fig. 6 displays the traditional chaos bandwidth and chaos flatness as a function of the feedback ratio for three different bias currents of 30 mA, 50 mA and 60 mA. We can see that the initial feedback ratios for different bias currents are different. This is because the laser with high bias current needs a high feedback ratio to trigger the laser into a chaotic dynamic [36]. From Fig. 6, we can see that chaos bandwidth increases monotonically with increasing feedback ratio for all three bias currents. For the bias current of 30 mA, the bandwidth of chaos almost increases linearly with the feedback ratio. However, for the bias current of 50 mA, the bandwidth increases rapidly between the feedback ratio of ~ 0.17 and ~ 0.28. Further increase of the feedback ratio see the rate of the bandwidth increasing decreases. For the bias current of 60 mA, the chaos bandwidth increases with the increasing feedback ratio, but the rate is slower than that of 30 mA. When the feedback ratio is below 0.49, the chaos bandwidths for the bias current of 50 mA are higher than those for the bias current of 30 mA at the same feedback ratio. This is easily understood because the chaos bandwidth is proportional to the relaxation oscillation frequency of the laser, and the relaxation oscillation frequency is proportional to the square root of the bias current minus the threshold current. However, for feedback ratios above 0.49, the chaos bandwidths for the bias currents of 30 mA, 50 mA and 60 mA are similar. This may be due to the bandwidth limitation of the cable connected between the photodetector and the power spectrum analyzer. In Fig. 6(a), we also note that the bandwidth at the feedback ratio around 0.30 for the bias current of 60 mA is smaller than that for the bias current of 50 mA, which is counterintuitive. To understand the reason, the power spectra of chaos at the same feedback ratio of 0.30 but different bias currents, as shown in Fig. 7, were checked. The power spectrum at 60 mA is typical of power spectrum of chaos obtained by optical feedback, where a peak appears near the relaxation oscillation frequency. However, for the power spectrum generated at 50mA, less power is concentrated around the relaxation oscillation frequency. Therefore, the chaos bandwidth at the bias current of 50 mA is slightly higher than that at the bias current of 60 mA.

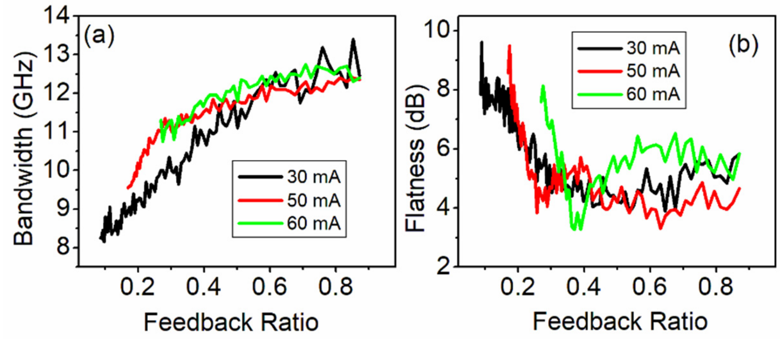

Fig.6 (a) The bandwidth and (b) flatness, of chaos as a function of the feedback ratio when the bias currents are 30 mA, 50 mA and 60 mA.

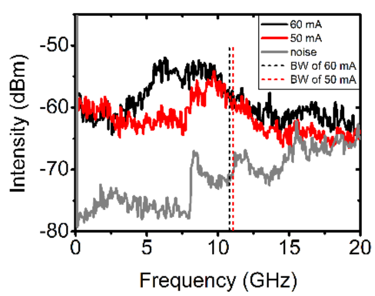

Fig.7 The power spectra of the laser at the feedback ratio of 0.30 with the bias currents of 50 mA and 60 mA. The dash lines correspondent to their bandwidths.

Figure 6(b) displays the chaos flatness as a function of the feedback ratio with the bias current of 30 mA, 50 mA and 60 mA. For the bias currents of 30 mA, the flatness suddenly drops, and then oscillates around ~ 7.7 dB between the feedback ratio of ~ 0.09 and ~ 0.16. Further increases in the feedback ratio, see the flatness further reduced until the feedback ratio reaches ~0.40. For the feedback ratio of ~ 0.40 to ~ 0.69, the flatness again stabilizes at a value of ~ 4.4 dB. Further increases the feedback ratio, see the flatness increases again. For the bias current of 50 mA, the flatness first falls rapidly to ~ 4.2 dB, and then quickly increases to ~ 5.4 dB. When the feedback ratio increases from ~ 0.39 to ~ 0.64, the flatness slowly decreases again, and has been reduced to ~ 3.3 dB. Further enhancement of the feedback ratio results in the flatness slowly increases again. For the bias current of 60 mA, the flatness first falls rapidly to ~ 3.2 dB at the feedback ratio of ~ 0.37, and then quickly increases to ~ 6.1 dB at the feedback ratio of ~ 0.6. Further increases the feedback ratio cause the flatness to fluctuate and decrease. From Figs. 5 and 6(b), we also note that the higher bias current, the better the flatness that can be achieved at the optimal feedback ratio, which is defined as a feedback ratio when the flatness of RF spectra of the chaotic output from the DM lasers reaches its minimum under a given bias current, but the narrower range of the feedback ratio that can be used to obtain a relatively flat spectrum.

4. Conclusion

Characterization of chaos generated in a discrete-mode laser with only optical feedback has been studied. The experimental results show that a flat broadband chaos with the traditional bandwidth of 12.1 GHz and effective bandwidth of 10.6 GHz and flatness of 2.8 dB can be obtained in the DM laser with only optical feedback when the bias current is 70 mA and the feedback ratio is ~ 0.68. The reason of achieving flat chaos in the DM laser with only optical feedback is due to the inherent cavity design of DM lasers, where the DM laser can be seen as a multimode laser with a band-pass filter, and mode competition is considered as the physical origin of generating flat chaos [24]. The experimental results also show that the DM laser with higher bias current can achieve better flatness at the optimal feedback ratio, however, the range of the feedback ratio for obtaining a relatively flat spectrum is smaller. This demonstration offers opportunities for designing simple structure of generating flat broadband chaos, which will benefit the applications of chaos.

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Disclosures

The authors declare no conflicts of interest.

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